# PREDICTIONS AND MEASUREMENTS OF NON-AXISYMMETRIC TURBULENT DIFFUSION **IN AN ANNULAR CHANNEL**

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Abstract-Measurements have been made of radial and circumferential diffusion of a tracer gas in fully developed flow in an annulus. The Reynolds number used was  $1.6 \times 10^5$  and the working fluid was air  $(Sc = 1.0)$ .

A predrction method is also described using a flow model based on the van Driest mixing length theory. Using a turbulent Schmidt number of unity, comparisons of the predictions and measurements show best agreement when the ratio of circumferential to radial mass diffusivity is chosen at the constant value of two throughout the flow.

#### NOMENCLATURE

- $A^+$ , constant in van Driest mixing length equation;
- 
- specific heat at constant pressure;
- c, concentration;<br> $C_p$ , specific heat at <br> $K$ , mixing length c mixing length constant;
- $k_h$ thermal conductivity of the fluid;
- $\frac{l}{l^+}$ mixing length ;
- non-dimensional mixing length  $[l(\tau_w/\rho)^{1/2}/v];$
- Nu, local Nusselt number  $(h \cdot 2r_e/k_h)$ ;<br>*Pr*, Prandtl number  $(v/\alpha)$ ;
- *Pr*, Prandtl number  $(v/\alpha)$ ;<br>*Pr*<sub>t</sub>, turbulent Prandtl num
- $Pr_t$ , turbulent Prandtl number  $(\varepsilon_m/\varepsilon_{hr});$ <br>q, heat flux;
- q, heat flux;<br> $r, r_e$ , radius and
- radius and hydraulic equivalent radius;
- $r_{ml}, r_{mt}$ , radius of maximum velocity for laminar and turbulent flow;
- $r^+$ non-dimensional radius  $[r(\tau_w/\rho)^{1/2}/v]$ ;
- $R_1, R_2$ , inner and outer radii of the annulus;<br>*Re*, Reynolds number  $(u_h \cdot 2r_s/v)$ ;
- Re, Reynolds number  $(u_b \cdot 2r_e/v)$ ;<br>Sc, Schmidt number ;
- $Sc$ , Schmidt number ;<br>t, T, temperature and r
- temperature and non-dimensional temperature;
- $u,$  velocity;<br> $u^+,$  non-dime
- non-dimensional velocity  $[u/(\tau_w/\rho)^{1/2}]$ ;
- $u',$ velocity fluctuations in the axial direction;
- $v'$ , velocity fluctuations in the radial direction;
- $w'$ , velocity fluctuations in the tangential direction;
- x, distance along the duct ;
- $y$ , distance from the wall;
- $y_c$ , distance from the wall to the point of maximum velocity ;
- $y_i$ , distance from the wall over which van Driest mixing length is used;

# $y^*$ , non-dimensional distance from the wall  $[y(\tau_w/\rho)^{1/2}/v].$

Creek symbols

- $\alpha$ , molecular diffusivity of heat  $(k_h/\rho C_p)$ ;<br> $\beta$ , radius ratio  $(R_1/R_2)$ ;
- 
- $\beta$ , radius ratio  $(R_1/R_2)$ ;<br> $\varepsilon_h$ , eddy diffusivity of hea eddy diffusivity of heat ;
- $\varepsilon_m$ , eddy diffusivity of momentum;<br> $\theta$ , angle;
- angle;
- $\mu$ , dynamic viscosity;<br>  $\nu$ , kinematic viscosity
- 
- v, kinematic viscosity  $(\mu/\rho)$ ;<br>  $\xi$ , ratio of the outer to inner ratio of the outer to inner wall shear stresses;
- $\rho$ , density;
- $\tau$ , shear stress.

## Subscripts

- b, bulk mean;
- $i,$  inner flow region;<br>in, inlet;
- inlet ;
- $m$ , position of maximum velocity;
- $\alpha$ , outer flow region;
- r, reference value ;
- wall value ; w.
- $\theta$ , in the tangential direction;
- $\infty$ , fully developed value.

### 1. **INTRODUCTION**

**THERE** are many situations in practice where the turbulent diffusion of heat takes place in more than one direction. In the fuel rod cluster of a nuclear reactor, for example, the situation is not axisymmetric and diffusion will occur in non-radial directions. It is usual in predictions of turbulent flows to assume the eddy viscosity to be locally isotropic but there is a good deal of evidence to show that this is not true. However, there is no uniformity of opinion on the relative diffusion rates in non-axisymmetric situations. Most previous work has been carried out in plain tubes. The first experimental work to be published with nonaxisymmetric boundary conditions was that of Hall

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and Hashimi [1]. Using a mass-transfer analogy with nitrous oxide diffusing through air, they measured concentration profiles downstream of a 25.4mm dia porous graphite plug set flush in the wall of a 50.8 mm dia tube. Reynolds [2] presented an analysis for hydrodynamically and thermally fully developed heat transfer in a circular tube with variable circumferential heat flux based on the assumption of equal radial and circumferential diffusivities over the entire crosssection. The same assumption was used by Sparrow and Lin  $\lceil 3 \rceil$  who produced a similar analysis to that of Reynolds.

Experimentalinvestigations with non-axisymmetric boundary conditions were carried out by Black and Sparrow [4] in an attempt to verify the validity of the analysis of Reynolds and Sparrow and Lin. It was shown that the analysis produced the correct trends of variation but the magnitudes were over-predicted especially at high Reynolds numbers. When a value of 10 for the ratio of circumferential to radial eddy diffusivity over the entire cross-section was used. improved correlation of the air stream temperature variations was observed but only in the wall region because, as would be expected, in the middle region the damping of the variations by the relatively high circumferential diffusion was more marked. Quarmby and Anand [S], also, made use of the analogy between heat and mass transfer by diffusing nitrous oxide from both a line source and a wall patch in a smooth tube. Their analysis, based on equal diffusivities in the two directions, showed good agreement with the experiments. Quarmby and Quirk [6]. however. later concluded that the ratio of circumferential to radial eddy diffusivity is a simple function of radius and has a high value near the wall but appears to be unity over the greater part of the core flow.

It appears from the above references that the calculations of concentrations or temperature distribution in the circular tube geometry with nonaxisymmetric boundary conditions are insensitive to variations in the ratio of the two diffusivities. It was then thought that the annular geometry, in which distinct anisotropy of the turbulent structure had been measured by Heikal et al. [7], would constitute a better test of the validity of any particular circumferential diffusion model.

Walker and Robinson [8] conducted an experimental and theoretical study of the problem of nonaxisymmetric, three dimensional mass transfer into fully developed turbulent flow in a concentric annulus. They employed different models of the ratio of circumferential to radial diffusivity in the analysis which had been suggested previously for tube flows. Comparisons between the predicted and measured concentration profiles at various axial and circumferential locations indicated the anisotropic nature of the eddy diffusivity. However, they concluded that the solution was insensitive to the degree of anisotropy in the wall region. They suggested that a constant mean value of two for the eddy diffusivity ratio across the whole section would probably give good fit between theory and experiment over a wide range of Reynolds numbers.

Despite all this work it is clear that both the magnitude and variation of the ratio of the tangential and radial diffusivities are still uncertain.

The object of the present work is to investigate this phenomenon in fully developed turbulent flow in an annulus by comparing concentration measurements with theoretical predictions using various arbitrary choices for the eddy diffusivity variation.

#### 2. EXPERIMENTAL APPARATLIS AND PROCEDURE

The tests were carried out in an annulus ofinner and outer radii 12.7 and 50.8 mm and total length of 6.0m giving an overall length to equivalent diameter ratio of about 79. A detailed description of the apparatus is given in Heikal et al. [7]. The tracer gas (nitrous oxide) was injected at 49 equivalent diameters from entry. The injectors were as shown in Fig.  $1(a)$  and (b). Using



FIG. 1. Injector arrangements and sampling probe.

an IR gas analyser, the concentration profiles were measured at different axial positions downstream of the injector at 20" intervals relative to the injector. The analyser was adjusted and calibrated before each test in case any drift in the electronic components took place and the sampling rate was kept constant at 1000 cm<sup>3</sup>/min throughout the test. The local concentration at each point was measured by drawing a sample of the nitrous oxide-air mixture through a probe made of 1.6mm dia stainless steel hypodermic tubing as shown in Fig.  $1(c)$ .

In view of the large response time of the measurement system, it was found that a sampling time of about 180 s was required before a steady reading **could**  be obtained.

The initial condition used in the theoretical prediction was taken as the first concentration profile which was measured at one equivalent diameter downstream of the injector. The situation at the injector was therefore not a significant factor in the comparisons between the measurements and predictions.

#### 3. THEORY

Whilst the experiments were carried out on mass transfer, the main objective of the work is the study of heat diffusion. The analysis is therefore carried out on the thermal energy equation which, for a fully developed turbulent flow including the eddy diffusivity concept, can be written as follows:

$$
u\frac{\partial t}{\partial x} = \frac{1}{r}\frac{\partial}{\partial r}\left[r(\alpha + \varepsilon_{hr})\frac{\partial t}{\partial r}\right] + \frac{1}{r^2}\frac{\partial}{\partial \theta}\left[(\alpha + \varepsilon_{h\theta})\frac{\partial t}{\partial \theta}\right] \tag{1}
$$

where :

$$
\varepsilon_{hr}\frac{\partial t}{\partial r}=-\overline{v't'}\ \text{ and }\ \varepsilon_{hu}\frac{\partial t}{\partial \theta}=-\overline{w't'}.
$$

It is assumed that viscous dissipation and axial diffusion are negligible and that the fluid physical properties are constant. The eddy diffusivity of heat in the radial direction is obtained from that of momentum using a turbulent Prandtl number, while the tangential diffusivity is calculated from the radial by assuming a ratio between the two.

#### *The eddy viscosity turbulence model*

The inner and outer parts of the flow were each divided into two regions, namely "the wall region" and "the core region".

*The wall region* ( $y_1 > y > 0$ ). In this region the eddy diffusivity was derived from a mixing length in accordance with Prandtl hypothesis:

$$
\varepsilon_m = l^2 \bigg( \frac{\mathrm{d}u}{\mathrm{d}y} \bigg). \tag{2}
$$

The mixing length expression used was that of van Driest:

$$
l = Ky[1 - \exp(-y^+/A^+)]
$$
 (3)

where  $K = 0.4$  and  $A^{+} = 26$ .

The extent of this region in the inner part of the flow was taken to be 0.35 of the distance between the wall and the radius of maximum velocity since it was found to give the best agreement with the experimental results.

The shear stress can be expressed as :

$$
\tau = (\mu + \rho \varepsilon_m) \frac{\mathrm{d}u}{\mathrm{d}y} \tag{4}
$$

where :

$$
\varepsilon_m \frac{\mathrm{d} u}{\mathrm{d} y} = -\overline{u'v'}
$$

from a simple force balance it can be shown that:

$$
\frac{\tau}{\tau_{w_i}} = \frac{R_1(r_m^2 - r^2)}{r(r_m^2 - R_1^2)} \text{ for } R_1 < r < r_m \tag{5}
$$

and

$$
\frac{\tau}{\tau_{w_o}} = \frac{R_2(r^2 - r_m^2)}{r(R_2^2 - r_m^2)} \text{ for } r_m < r < R_2 \tag{6}
$$

whilst

$$
\frac{\tau_o}{\tau_i} = \xi = \frac{R_1(R_2^2 - r_m^2)}{R_2(r_m^2 - R_1^2)}\tag{7}
$$

substituting  $(2)$  and  $(5)$  into  $(4)$  we obtain:

$$
\tau_{w_i} \left[ \frac{R_1 (r_m^2 - r^2)}{r (r_m^2 - R_1^2)} \right] = \rho l^2 \left( \frac{\mathrm{d} u}{\mathrm{d} y} \right)^2 + \mu \frac{\mathrm{d} u}{\mathrm{d} y} \tag{8}
$$

Non-dimensionalising and rearranging we get:

$$
l_i^{+2} \left(\frac{du_i^{+}}{dr_i^{+}}\right)^2 + \left(\frac{dy_i}{dr_i^{+}}\right) - z_i^{+} = 0
$$

where

$$
z_i^+ = \frac{R_{1i}^+(r_{mi}^2 - r_i^2)}{r_i^+(r^{+2} - R_{1i}^{+2})}
$$

solving for  $(du_i^+/dr_i^+)$  we obtain the result:

$$
\frac{du_i^+}{dr_i^+} = \frac{-1 + (1 + 4l_i^+ z_i^+)^{1/2}}{2l_i^+ z}.
$$
 (9)

A similar equation was obtained for the outer wall region the extent of which was obtained by matching the value of the eddy viscosity in the core, which was obtained from the inner wall region at  $y = y_t$ .

The integration of equation (9) was done numerically using the Runge-Kutta technique and the step length was varied so as to give five figure accuracy in the integrals.

*The core region*  $(y_1 < y < y_c)$ . In this region it is assumed that the eddy viscosity is constant throughout this region and that its value can be obtained from the wall region at  $y = y_i$ .

Non-dimensionalising equation (4) and (5), rearranging and integrating we obtain :

 $\mathcal{L}$ 

$$
u_i^+ = \frac{R_{1i}^+ \left(\frac{v}{v+\varepsilon_{mc}}\right)}{(r_{mi}^{+2}-R_{1i}^{+2})} \left(r_m^{+2} \ln r_i^+ - \frac{1}{2}r_i^{+2}\right) + M_i
$$

where  $M_i$  is a constant of integration and can be determined at  $r_i^+ = R_{1i}^+ + y_{li}^+$ .

The radius of maximum velocity was obtained by matching the inner and outer velocity profiles at an assumed point and the solution was iterated to obtain the correct point.

*General analysis for the solution of the thermal energy equation* 

The non-dimensional form of equation (1) is :

$$
u_i^+ u_{mi}^+ \frac{\partial T}{\partial R} = \frac{1}{r_i^+} \frac{\partial}{\partial r_i^+} \left[ r_i^+ \left( \frac{1}{Pr} + \frac{\varepsilon_{hr}}{v} \right) \frac{\partial T}{\partial r_i^+} \right] + \frac{1}{r_i^+^2} \frac{\partial}{\partial \theta} \left[ \left( \frac{1}{Pr} + \frac{\varepsilon_{h\theta}}{v} \right) \frac{\partial T}{\partial \theta} \right] \tag{10}
$$

where  $R = \frac{m}{v}$ 

and

$$
T = \frac{t - t_{\text{in}}}{\left(\frac{q_{\text{wr}}}{\rho C_p u_m}\right)}
$$

(for a specified wall heat flux).

$$
T = \frac{t - t_{\text{in}}}{(t_w - t_{\text{in}})_r}
$$

(for a specified wall temperature).

Combining the non-dim'ensionalised forms of (4) and



FIG. 2. Variation of the radius of maximum velocity with Reynolds **numbers** 

(5) yield the result:

$$
\frac{\varepsilon_{hr}}{v} = \frac{1}{Pr_t} \left[ \frac{R_{1i}^+(r_{m}^{+2} - r_i^{+2})}{r_i^+(r_{m}^{+2} - R_{1i}^{+2})} \frac{\mathrm{d}r_i^+}{\mathrm{d}u_i^+} - 1 \right]. \tag{11}
$$

Equation (10) can be written :

$$
\frac{\partial T}{\partial R} = A_1 \frac{\partial^2 T}{\partial r_i^2} + B_1 \frac{\partial T}{\partial r_i^2} + D \frac{\partial^2 T}{\partial \theta^2}
$$
 (12)

where

$$
A_1 = \frac{\left(\frac{1}{Pr} + \frac{\varepsilon_{hr}}{v}\right)}{u_i^+ u_m^+}
$$

$$
B_1 = \frac{\left[\frac{1}{r_i^+} \left(\frac{1}{Pr} + \frac{\varepsilon_{hr}}{v}\right) + \frac{\partial}{\partial r_i^+} \left(\frac{1}{Pr} + \frac{\varepsilon_{hr}}{v}\right)\right]}{u_i^+ u_m^+}
$$

and

$$
D=\frac{\frac{1}{r_i^+}\left[\frac{1}{Pr}+\frac{\varepsilon_{h0}}{v}\right]}{u_i^+ u_{mi}^+}.
$$

and  $\varepsilon_{hr}/v$  is obtained from equation (11).

Equation (12) was solved using a Crank-Nicholson implicit finite difference method with an arbitrary weighting factor and 140 equal increments of  $r_i^+$ considering the term  $D(\partial^2 T/\partial \theta^2)$  as a constant. The correct value of this term was then obtained from the resulting temperature field description and iterating the solution. The weighting factor was taken at 0.9, although other values were tried and found to make little difference. A typical run time on the UMRCC CDC 7600 was 60s for 160 steps downstream (40 diameters) for the problem considered.

#### 4. **DISCUSSION**

#### 4.1 *Resultsfor axisymmetric situations*

Before considering the degree of success in the prediction of non-axisymmetric situation it is first necessary to ensure that the prediction method is compatible with the considerable amount of data which exists for axisymmetric cases. It is particularly important to ensure that the theory gives a correct description of the radial diffusivity.

Figure 2 shows comparisons of predictions of the

radius of maximum velocity with the measurements of Quarmby [9] which were obtained using Preston tubes. The agreement is better at higher radius ratios  $(R_1/R_2)$ . It has been suggested that shear stress measurements using Preston tubes may not be reliable on the outside of the core tube at low radius ratios.

At higher Reynolds numbers the radius of maximum velocity becomes independent of the value of Reynolds number and is well correlated by

$$
\frac{r_m - R_1}{R_2 - r_m} = \left(\frac{R_1}{R_2}\right)^{0.323}
$$

Figure 3 shows similar comparisons of inner and outer velocity profiles and the agreement is good. The comparisons are not as sensitive to the shear stress measurements as those discussed above.

Shear stress profiles are compared on Fig. 4. The results for the radius ratio of0.25 were obtained on the same apparatus using hot wires and are reported in 171. Very good agreement is shown between the predictions and measurements.

The heat-transfer predictions are compared with results again taken from [7] and with those of other workers for  $Pr<sub>t</sub> = 1$  on Fig. 5 and once again it is clear that the prediction is in good agreement over a wide range of Reynolds numbers and radius ratios. Although not shown the predicted Nusselt numbers in the thermal entrance region also agreed well with the measurements of [7].

To summarise, the results discussed so far show that the turbulence model **used** for fully developed flow in an annulus is compatible with the available previous work both experimental and theoretical. The program is useful for calculating the effect of axial variations of wall heat flux or temperature. Whilst there is no experimental data for the annulus, the theoretical predictions for sinusoidal, exponential and linear variation in the axial direction of heat flux on the inner wall showed very similar trends to the experimental work of Hall and Price [10] in a round tube.

#### 4.2 *Non-axisymmetric situations*

*Two* situations were examined and the arrangements are shown on Figs. l(a) and (b). Arrangement



FIG. 3. Non-dimensional velocity profiles in an annulus of 0.1067 radius ratio.



FIG. 4. Comparison of shear stress profiles.



FIG. 5. Fully developed Nusselt numbers in annuli of different radius ratios.

(a) was chosen with the intention ofexamining the core region of the flow, whereas it was hoped that arrangement (b) would throw some light on the situation near to the wall.

For mass-transfer predictions, of course, where the eddy diffusivity of mass is obtained from that of momentum it is necessary to assume a turbulent Schmidt number in place of the turbulent Prandtl number. Betts and Hatton [11] showed that there is considerable uncertainty regarding turbulent Schmidt numbers in the radial direction but the value of unity appears reasonable and was used in these calculations.

For the prediction of the non-axisymmetric experiments the procedure adopted was to measure radial concentration profiles at different angular and axial positions downstream of the injector and to compare the results with predictions using different assumptions regarding the ratio of the circumferential to radial diffusivities. The initial profiles were measured and fed into the program as starting values for the prediction. The assumptions were either:

(i) a constant value throughout the flow;



FIG. 6(a). Comparison of concentration profiles along the annulus  $(x/2re = 1.5)$ .



FtG. 6(b). Comparison of concentration profiles along the annulus  $(x/2re = 4.5)$ .



FIG. 6(c). Comparison of concentration profiles along the annulus  $(x/2re = 6.5)$ .



FIG. 6(d). Comparison of concentration profiles along the annulus  $(x/2re - 9.0)$ 

(ii) a constant value in the core region together with a linearly increasing variation to the wall from some chosen position near to the wall. The slope of the linear variation and the position from which it started were chosen arbitrarily.

For arrangement (a) Figs.  $6(a)$ - $(d)$  show radial concentration profiles at different angular positions and at different stations downstream. Three constant values of  $\varepsilon_0/\varepsilon_r$ , were used for comparison in the prediction method. The best fit is obtained with a value of 2.0.

For arrangement (b) the results are not so clear. Figures  $7(a)$ -(c) show the profiles and in general the results, whilst not contributing further evidence to reinforce the choice of the value of 2.0, they do show that the choice of 2.0 gives as good comparisons as could be expected.

Further predictions were made using variations of the ratio as discussed above in assumption (ii). for example with the value of 5 at the wall varying to 1, 2 or 3 in the stream over a width of 0.3 of the distance



FtG. 7(a). Comparison of concentration profiles along the annulus  $(x/2re = 3.0)$ .



FIG. 7(b). Comparison of concentration profiles along the annulus  $(x/2re = 7.5)$ .



FIG. 7(c). Comparison of concentration profiles along the annulus ( $x/2re = 13.5$ ).

between the wall and the position of maximum velocity. No significant difference was obtained from using a constant value throughout. The differences in the predicted profiles were so small that they could not be shown on the scale of Figs. 6 and 7.

#### 5. CONCLUSIONS

1. A prediction method is proposed for mass diffusion or heat transfer in fully developed nonaxisymmetric turbulent flow in an annulus. The turbulence model used to obtain the radial eddy diffusivity used a constant value in the core region with a van Driest mixing length variation near the wall. Good agreement was obtained with available experimental results, for example shear stress, velocity profiles and friction factor.

2. The diffusion experiments show that the turbulence structure in the central region is clearly anisotropic.

3. The predictions show that the development of concentration profiles is insensitive to the variation in the ratio of the eddy diffusivities close to the wall.

4. The best agreement between the predictions and measurements results from using a constant ratio of  $\varepsilon_{\nu}/\varepsilon_{r}$  of 2 over the whole cross section.

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#### PRECISION ET MESURE DE LA DIFFUSION TURBULENTE NON AXISYMETRIQUE DANS UN CANAL ANNULAIRE

Résumé-On effectue des mesures de diffusion radiale et circonférentielle d'un gaz traceur dans un écoulement pleinement développé dans un espace annulaire. Le nombre de Reynolds est égal à  $1.6 \times 10^5$  et le fluide est l'air  $(S<sub>c</sub> = 1,0)$ . Une méthode de calcul est décrite qui utilise un modèle d'écoulement basé sur la théorie de longueur de mélange de Van Driest. Prenant un nombre de Schmidt turbulent égal à l'unité, des comparaisons entre calculs et mesures montrent un meilleur accord quand le rapport des diffusivitiés massiques circonférentielle et radiale est choisi égal à deux, valeur constante dans tout le fluide en écoulement.

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#### BERECHNUNG UND MESSUNG UNSYMMETRISCHER TURBULENTER DIFFUSION IN EINEM KREISRINGKANAL

Zusammenfassung---Es wurde die Diffusion eines Spurengases in der voll entwickelten Strömung in einem Kreisringkanal in radialer und in Umfangsrichtung gemessen. Die Reynolds-Zahl war 1.6·10<sup>3</sup> und das strömende Fluid Luft ( $Sc = 1.0$ ). Eine Berechnungsmethode wird ebenfalls beschrieben, die von einem Strömungsmodell ausgeht, welches auf der Mischungswegtheorie nach van Driest basiert. Rechnung und Messung zeigen die beste Übereinstimmung, wenn bei einer turbulenten Schmidt-Zahl von eins für das Verhältnis des Diffusionskoeffizienten in Umfangsrichtung zu dem in radialer Richtung für die gesamte Strömung des Wert zwei angenommen wird.

#### РАСЧЕТ И ИЗМЕРЕНИЕ НЕОСЕСИММЕТРИЧНОЙ ТУРБУЛЕНТНОЙ ДИФФУЗИИ В КОЛЬЦЕВОМ КАНАЛЕ

Аннотация — Проведены измерения радиальной и кольцевой диффузии меченого газа в полностью развитом потоке в кольцевом канале. Число Рейнольдса составляло 1,6 × 10<sup>5</sup>, в качестве рабочей жидкости использовался воздух ( $Sc = 1,0$ ). В статье описывается также метод расчета с помощью модели течения, построенной на основании теории длины смешения Ван Дриста. При турбулентном числе Шмидта, равном единице, результаты расчета хорошо согласуются с данными измерений в том случае, если отношение коэффициентов кольцевой и радиальной диффузии принимается как константа, равная 2, во всей области течения.